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Publication date:
1994

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for
omic Research

Discussion paper

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TILBURG UNIVERSITY

Center
for
Economic Research

No. 9424

DIGRAPH COMPETITIONS AND COOPERATIVE GAMES

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February 1994

ISSN 0924-7815

Digraph Competitions and Cooperative Games*

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February 1994

*The authors would like to thank Vincent Feltkamp, Rob Gilles and Stef Tijs for their useful remarks.

†This author is financially supported by the Netherlands Organization for Scientific Research (NWO), grant 450-228-022

Abstract

A dominance structure or competition between a set of players can be modelled by means of a *directed graph* (N, D). Here, the set of nodes N represents a set of players, and $(i, j) \in D$ means that player i dominates or defeats player j. We introduce the *conservative score mapping* which assigns to every digraph a *cooperative game with transferable utilities*. Solution concepts for these score games can be interpreted as a way to evaluate the underlying dominance structure or competition, i.e., as a way to rank the players.

We provide characterizations of the score mapping and of the subclass of score games. It is shown that a score game is *convex*, and its *core* is equal to the convex hull of the set of *score vectors* corresponding to those subgraphs in which each player that is defeated in the original digraph now is defeated exactly once. Consequently, all *marginal vectors* are such score vectors. However, in general the converse need not hold, and the class of digraphs for which it does is characterized.

Furthermore, the *Shapley value* of a score game, which by definition is the mean of all marginal vectors, turns out to be also the mean of all score vectors.

1 Introduction

A situation in which a set of players can obtain certain payoffs by cooperation can be represented by a *cooperative game with transferable utilities* – or simply a TU-game – being a pair (N, v) where N denotes the set of players and the *characteristic function* $v: 2^N \to \mathbb{R}$ is such that $v(\emptyset) = 0$. In the sequel we take the set N to be fixed and therefore represent each TU-game on N by its characteristic function. We denote the set of all TU-games on N by $\mathcal{G}^N$. If we model a cooperative situation in this way then the players only differ with respect to their contributions to the values of the various coalitions but further they are assumed to be socially symmetric. In recent literature models have been developed in which the players are part of a social organization structure which limits the possibilities of coalition formation. In some of these models this social structure is represented by a *graph*. As an example we mention the models in which there are limited communication possibilities between the players which are represented by an *undirected graph*. It is assumed that players can only cooperate if they can communicate with one another. For this we refer to Myerson

(1977), Owen (1986), Borm, Owen and Tijs (1992), and the survey paper by Borm, van den Nouweland and Tijs (1993). Another approach introducing social asymmetries between the players can be found in, e.g., Gilles, Owen and van den Brink (1992) and Faigle and Kern (1993). Here the players are part of some hierarchical organization which is represented by a *directed graph*. It is assumed that players can only cooperate if they have permission from their superiors in the hierarchy.

In all models mentioned above a directed or undirected graph limits the possibilities of coalition formation in a TU-game. From a TU-game and a graph another TU-game is derived taking into account the limited possibilities of cooperation. Of course, the position of a player in the graph affects his role in the modified game.

In this paper we will derive a TU-game from just a graph. Such a TU-game can be seen as a *social power measure* in the sense that the value of a coalition is a measure of the importance of the coalition in the social structure represented by the graph. We concentrate on directed graphs.

A directed graph or *digraph* is a pair (N, D) where N is a finite set of nodes and $D \subset N \times N$ is a binary relation on N. Such a digraph can represent various hierarchical organizations. Above we already mentioned the interpretation as a permission structure in which there are players that need permission from their superiors before they can cooperate. A digraph also can be interpreted as an exchange economy such that the nodes represent the agents that participate in the economy and the fact that agent i dominates agent j means that agent i sets the conditions under which trade between agent j and himself will take place. (For example, he sets the prices under which he exchanges commodities with agent j.) This interpretation is considered in van den Brink and Gilles (1993). Finally we mention two closely related interpretations of digraphs that can be found in, e.g., Rubinstein (1980) and Laffont, Laslier and Le Breton (1993). One is the interpretation of a digraph as a *preference relation* such that the nodes represent alternatives between which an agent can choose. The fact that alternative i dominates alternative j then means that an agent or group of agents prefers i to j when comparing these alternatives pairwise. The other is the interpretation of a digraph as a *competition* between players or teams. Then the nodes represent the players or teams that participate in the competition and $(i, j) \in D$ means that player i has won the match he played against player j.

In the sequel we do not want to fix the specific context. The various results are formulated in a general setting. In section 2 we introduce and axiomatize a specific mapping that assigns a (convex) TU-game to every digraph on the set N. The games that can be derived in this way are called *conservative score games* and form a subclass of $\mathcal{G}^N$. We provide a characterization of this subclass. It is seen that a conservative score game is convex and in section 3 we show that the core of such a game coincides with the convex hull of the set of score vectors corresponding to those subgraphs of D in which each player that is dominated in D is dominated by exactly one other player. As a consequence all marginal vectors are such score vectors. The class of digraphs for which the converse also holds is characterized by means of the non-existence of so-called *anti-directed semi-circuits*. In section 4 we consider the Shapley value (Shapley (1953)) of a conservative score game and argue that it is equal to the *BG-measure* which is introduced in van den Brink and Gilles (1992) as a *power measure* for digraphs. It is shown that the Shapley value of a conservative score game, which by definition is the mean of all marginal vectors, is also the mean of the score vectors that are considered in section 3.

We conclude this section by presenting some concepts about digraphs that will be used in the sequel. We only consider *finite, irreflexive* digraphs. A digraph (N, D) is finite if N is finite and it is irreflexive if $(i,i) \notin D$ for every $i \in N$. We simply refer to these graphs as digraphs. In the sequel we assume the set N to be fixed and therefore we represent a digraph by its binary relation. The collection of all digraphs on N is denoted by $\mathcal{D}^N$.

Let $D \in \mathcal{D}^N$. For $E \subset N$, the nodes in

$$S_D(E) := \{j \in N \mid \text{ there is an } i \in E \text{ such that } (i,j) \in D\}$$

are called the *successors* of E in D. For node $i \in N$ the nodes in

$$P_D(i) := \{j \in N \mid (j,i) \in D\}$$

are called the *predecessors* of node i in D. (We will often omit the subscript D.) The *score measure* on N is the function $\sigma\colon \mathcal{D}^N \to \mathbb{R}^N$ that assigns to every node

i in digraph D the number of successors of i. A digraph A is a *single predecessor digraph* if $\#P_A(i) \leq 1$ for all $i \in N$. A single predecessor digraph in $D \in \mathcal{D}^N$ is a single predecessor digraph $A \subset D$ such that $P_A(i) \neq \emptyset$ if and only if $P_D(i) \neq \emptyset$. The collection of all single predecessor digraphs in D is denoted by $\mathcal{A}_D$.

2 The conservative score game

In this section we introduce a specific mapping that assigns a cooperative game with transferable utilities to every digraph on the set of nodes N. The set of players in such a TU-game corresponds to the nodes of the digraph while the characteristic function is given in the following definition.

Definition 2.1 *The* **conservative score mapping** *on N is the mapping $v^c: \mathcal{D}^N \to \mathcal{G}^N$ given by*

$$v^c(D)(E) = \#\{j \in S(E) \mid P(j) \subset E\} \quad \textit{for all } E \subset N \textit{ and } D \in \mathcal{D}^N.$$

For $D \in \mathcal{D}^N$ the game $v^c(D)$ is called the *conservative score game* corresponding to D and it assigns to every coalition $E \subset N$ the number of successors of E that have no predecessors outside E. If we interpret this number as a measure of the importance of coalition E in the digraph then the conservative score game can be seen as a *social power measure* that measures the 'dominance power' of coalitions of nodes in a digraph. Using *unanimity games* u_T, which for every $T \in 2^N \setminus \{\emptyset\}$ are given by

$$u_T(E) = \begin{cases} 1 & \text{if } E \supset T \\ 0 & \text{else.} \end{cases}$$

it is easy to see that a conservative score game can be expressed as in the following lemma.

Lemma 2.2 *For every $D \in \mathcal{D}^N$ it holds that*

$$v^c(D) = \sum_{i \in S(N)} u_{P(i)}.$$

Example 2.3 Consider the digraph $D = \{(1,2),(3,2),(2,4),(3,4)\}$ on $N = \{1,\ldots,4\}$. Since $P(1) = P(3) = \emptyset$, $P(2) = \{1,3\}$ and $P(4) = \{2,3\}$ it follows from Lemma 2.2 that $v^c(D) = u_{\{1,3\}} + u_{\{2,3\}}$.

From the fact that each conservative score game can be expressed as a positive sum of unanimity games it follows that the conservative score game corresponding to $D \in \mathcal{D}^N$ is *convex* which means that

$$v^c(D)(E) + v^c(D)(F) \leq v^c(D)(E \cup F) + v^c(D)(E \cap F) \text{ for all } E, F \subset N.$$

Next we give four axioms on a mapping $v\colon \mathcal{D}^N \rightarrow \mathcal{G}^N$ that uniquely determine the conservative score mapping. The first axiom states that the value of the grand coalition N is equal to the total number of dominated nodes in the digraph.

Axiom 2.4 (Efficiency) *For every $D \in \mathcal{D}^N$ it holds that*

$$v(D)(N) = \#S(N).$$

The second axiom states that a node that does not dominate any other node in the digraph adds the value zero to every coalition in the corresponding game.

Axiom 2.5 (Dummy property) *For every $D \in \mathcal{D}^N$ and $i \in N$ with $S(i) = \emptyset$ it holds that*

$$v(D)(E) = v(D)(E \setminus \{i\}) \textit{ for all } E \subset N.$$

The third axiom states that a node that dominates all dominated nodes in the digraph is necessary for any coalition to obtain a non-zero payoff in the corresponding game.

Axiom 2.6 (Top property) *For every $D \in \mathcal{D}^N$ and $i \in N$ with $S(i) = S(N)$ it holds that*

$$v(D)(E) = 0 \textit{ for all } E \subset N \setminus \{i\}.$$

The fourth axiom states that if we consider the union of two or more digraphs on the same set of nodes which are such that each node is dominated in at most one of these digraphs then the corresponding game equals the sum of the games corresponding to the constituting subdigraphs. In order to formalize this idea we introduce the following definitions. A *partition* of $D \in \mathcal{D}^N$ is a collection $\mathcal{P} = \{D_1, \ldots, D_t\}$ of digraphs such that the following two conditions are satisfied:

- $\bigcup_{k=1}^{t} D_k = D$;
- $D_k \cap D_l = \emptyset$ for all $1 \leq k, l \leq t$, $k \neq l$.

A partition $\mathcal{P}$ of a digraph D is *independent* if each node is dominated in at most one digraph in this partition, i.e, if besides the two conditions stated above $\mathcal{P}$ also satisfies:

- $\#\{A \in \mathcal{P} \mid P_A(i) \neq \emptyset\} \leq 1$ for all $i \in N$.

Axiom 2.7 (Additivity over independent partitions) *For every $D \in \mathcal{D}^N$ and each independent partition $\mathcal{P}$ of D it holds that*

$$v(D) = \sum_{A \in \mathcal{P}} v(A).$$

The four axioms introduced above uniquely determine the conservative score mapping.

Theorem 2.8 *A mapping $v: \mathcal{D}^N \to \mathcal{G}^N$ is equal to the conservative score mapping if and only if it satisfies efficiency, the dummy property, the top property and additivity over independent partitions.*

PROOF

It can easily be verified that the conservative score mapping satisfies the four axioms. Now let $v: \mathcal{D}^N \to \mathcal{G}^N$ satisfy the four axioms and let $D \in \mathcal{D}^N$.

For every $j \in S(N)$ we introduce the digraph $D_j \in \mathcal{D}^N$ where

$$D_j := \{(i, j) \in D \mid i \in P(j)\}.$$

From efficiency of v it follows that $v(D_j)(N) = 1$. Let $E \subset N$.
If $E \supset P(j)$ then $S_{D_j}(i) = \emptyset$ for every $i \in N \setminus E$. The dummy property then implies that $v(D_j)(E) = v(D_j)(N) = 1$.
Otherwise, if $E \not\supset P(j)$, then $P(j) \cap (N \setminus E) \neq \emptyset$. Since $S_{D_j}(i) = \{j\} = S_{D_j}(N)$ for all $i \in P(j) \cap (N \setminus E)$, the top property implies that $v(D_j)(E) = 0$.
The collection $\{D_j\}_{j \in S(N)}$ is an independent partition of D. From additivity over independent partitions it then follows that

$$v(D)(E) = \sum_{j \in S(N)} v(D_j)(E) = \#\{j \in S(E) \mid P(j) \subset E\} = v^c(D)(E) \text{ for all } E \subset N.$$

□

The conservative score mapping assigns a TU-game to every digraph on N. However, not every TU-game can be the conservative score game corresponding to some digraph. The following proposition provides a necessary and sufficient condition for a TU-game to be the conservative score game corresponding to some digraph.

Proposition 2.9 *Let $v \in \mathcal{G}^N$. Then v is a conservative score game if and only if there exists a sequence $\mathcal{T} = (T_1, \ldots, T_t)$ of coalitions, and a sequence $\mathcal{I} = (i_1, \ldots, i_t)$ of players such that the following three conditions are satisfied:*

(i) $i_k \in N \setminus T_k$ *for all* $k \in \{1, \ldots, t\}$;

(ii) $i_k \neq i_l$ *for all* $k, l \in \{1, \ldots, t\}$, $k \neq l$;

(iii) $v = \sum_{T \in \mathcal{T}} u_T$.

PROOF

Only if

Suppose that v is the conservative score game corresponding to $D \in \mathcal{D}^N$. According to Lemma 2.2 it holds that $v = \sum_{i \in S(N)} u_{P(i)}$.
Let $S(N) = \{s_1, \ldots, s_t\}$. Taking $\mathcal{T} = (P(s_1), \ldots, P(s_t))$ and $\mathcal{I} = (s_1, \ldots, s_t)$, the conditions (i), (ii) and (iii) are satisfied.

If

Let $\mathcal{T} = (T_1, \ldots, T_t)$ and $\mathcal{I} = (i_1, \ldots, i_t)$ satisfy conditions (i), (ii) and (iii).
Let $D \in \mathcal{D}^N$ be given by $D = \bigcup_{k=1}^t D_k$ where $D_k := \{(j, i_k) \mid j \in T_k\}$ for every $k \in \{1, \ldots, t\}$. Then it readily follows that v is the conservative score game corresponding to D.

□

Thus Proposition 2.9 characterizes the class of all conservative score games on N. An alternative to the conservative score mapping is the *(optimistic) score mapping** which is the mapping $v^s\colon \mathcal{D}^N \rightarrow \mathcal{G}^N$ given by

$$v^s(D)(E) = \#S(E) \text{ for every } E \subset N \text{ and } D \in \mathcal{D}^N.$$

Thus the (optimistic) score game corresponding to $D \in \mathcal{D}^N$ assigns to every coalition $E \subset N$ the total number of successors of E, irrespective of the fact whether these successors have predecessors outside E or not. Thus this game can be seen as a generalization of the score measure that is given in section 1. For every $D \in \mathcal{D}^N$ the corresponding (optimistic) score game is the *dual game* of the corresponding conservative score game.

Lemma 2.10 *For every $D \in \mathcal{D}^N$ it holds that*

$$v^c(D)(N) - v^c(D)(N \setminus E) = v^s(D)(E) \textit{ for all } E \subset N.$$

PROOF
Let $D \in \mathcal{D}^N$ and $E \subset N$. Then

$$\begin{aligned} v^c(D)(N) - v^c(D)(N \setminus E) &= \#\{j \in S(N) \mid P(j) \subset N\} \\ &\quad -\#\{j \in S(N \setminus E) \mid P(j) \subset N \setminus E\} \\ &= \#S(N) - \#\{j \in S(N) \mid P(j) \cap E = \emptyset\} \\ &= \#\{j \in S(N) \mid P(j) \cap E \neq \emptyset\} = \#S(E) = v^s(D)(E). \end{aligned}$$

*The (optimistic) *score game* corresponding to a digraph is introduced in van den Brink and Gilles (1992).

□

The optimistic score mapping can be axiomatized by replacing the top property in Theorem 2.8 by a *dual top property* which states that for every $D \in \mathcal{D}^N$ and $i \in N$ with $S(i) = S(N)$ it holds that $v(D)(E) = v(D)(N)$ for all $E \ni i$.

Example 2.11 In this example we illustrate the independence of the four axioms stated in Theorem 2.8.

1. Let the mapping $v^1 \colon \mathcal{D}^N \to \mathcal{G}^N$ be given by

   $$v^1(D)(E) = \frac{1}{2} \cdot v^c(D)(E) \text{ for all } E \subset N \text{ and } D \in \mathcal{D}^N.$$

   This mapping satisfies all four axioms except efficiency. For the digraph of Example 2.3 it holds that $v^1(D)(N) = 1 < 2 = \#S(N)$.

2. Let the mapping $v^2 \colon \mathcal{D}^N \to \mathcal{G}^N$ be given by

   $$v^2(D)(E) = \#\{j \in S(E) \mid (P(j) \cup \{j\}) \subset E\} \text{ for all } E \subset N \text{ and } D \in \mathcal{D}^N.$$

   This mapping satisfies all four axioms except the dummy property. For the digraph of Example 2.3 it holds that $v^2(D)(\{2,3\}) = 0$ and $v^2(D)(\{2,3,4\}) = 1$ although $S(4) = \emptyset$.

3. Consider the score mapping $v^s \colon \mathcal{D}^N \to \mathcal{G}^N$ that is discussed above. This mapping satisfies all four axioms except the top property. For the digraph of Example 2.3 it holds that $v^s(D)(\{1\}) = 1$ although $S(3) = \{2,4\} = S(N)$.

4. Let the mapping $v^3 \colon \mathcal{D}^N \to \mathcal{G}^N$ for every $E \subset N$ and $D \in \mathcal{D}^N$ be given by

   $$v^3(D)(E) = \#\{j \in S(E) \cup S(S(E)) \mid P(j) \cup P(P(j)) \subset E\}.$$

   This mapping satisfies all four axioms except additivity over independent partitions. The two digraphs $D^1 = \{(1,2),(3,2)\}$ and $D^2 = \{(2,4),(3,4)\}$ form an independent partition of the digraph D of Example 2.3. However,

   $$v^3(D^1)(\{2,3\}) + v^3(D^2)(\{2,3\}) = 0 + 1 \neq 0 = v^3(D)(\{2,3\}).$$

3 The core of a conservative score game

In this section we consider the *core* of a conservative score game. The core of an arbitrary TU-game $v \in \mathcal{G}^N$ is given by

$$Core(v) = \left\{ x \in \mathbb{R}^N \;\middle|\; \begin{array}{c} \sum_{i \in N} x_i = v(N) \\ \text{and} \\ \sum_{i \in E} x_i \geq v(E) \text{ for all } E \subset N \end{array} \right\}.$$

As stated in the previous section each conservative score game is *convex*. In Shapley (1971) it is shown that the core of a convex game v corresponds to the convex hull of the *marginal vectors*, $\{m^\pi(v)\}_{\pi \in \Pi(N)}$, where $\Pi(N)$ denotes the collection of all permutations on N and for every $\pi \in \Pi(N)$ and $v \in \mathcal{G}^N$, $m^\pi(v) \in \mathbb{R}^N$ is given by:

$$m_i^\pi(v) = v(\{j \in N \mid \pi(j) \leq \pi(i)\}) - v(\{j \in N \mid \pi(j) < \pi(i)\}) \text{ for all } i \in N. \qquad (1)$$

Using this result we can characterize the core of a conservative score game. For every $D \in \mathcal{D}^N$ we define

$$M(D) = \{m^\pi(v^c(D)) \in \mathbb{R}^N \mid \pi \in \Pi(N)\},$$

being the collection of all marginal vectors of the conservative score game corresponding to D, and

$$\Sigma(D) = \{\{\sigma(A) \in \mathbb{R}^N \mid A \in \mathcal{A}_D\},$$

being the collection of score measures of all single predecessor digraphs in D.

Theorem 3.1 *For every $D \in \mathcal{D}^N$ it holds that*

$$Core(v^c(D)) = Conv(\Sigma(D)).$$

PROOF
Let $D \in \mathcal{D}^N$. The proof of the theorem consists of the following steps.

(a) We first prove that $M(D) \subset \Sigma(D)$ by showing that for every $\pi \in \Pi(N)$ there is an $A \in \mathcal{A}_D$ such that $m^\pi(v^c(D)) = \sigma(A)$. Therefore let $\pi \in \Pi(N)$. Then

$$m_i^\pi(v^c(D)) = \#\{j \in S_D(i) \mid \pi(h) \leq \pi(i) \text{ for all } h \in P_D(j)\} \text{ for all } i \in N.$$

Let $A \in \mathcal{D}^N$ be given by $A = \{(i,j) \in D \mid \pi(h) \leq \pi(i) \text{ for all } h \in P_D(j)\}$.

Since $A \subset D$ and $P_A(j) = \{i \in P_D(j) \mid \pi(i) \geq \pi(h) \text{ for all } h \in P_D(j)\}$ it is easy to see that $A \in \mathcal{A}_D$ and $\sigma(A) = m^\pi(v^c(D))$.

(b) Next we prove that $\Sigma(D) \subset Core(v^c(D))$ by showing that $\sigma(A) \in Core(v^c(D))$ for all $A \in \mathcal{A}_D$. Therefore let $A \in \mathcal{A}_D$.

By definition of a single predecessor digraph it holds that

$$\sum_{i \in N} \sigma_i(A) = \#S_D(N) = v^c(D)(N).$$

Further, $\sum_{i \in E} \sigma_i(A) = \#\{j \in S_D(E) \mid P_A(j) \cap E \neq \emptyset\} \geq \#\{j \in S_D(E) \mid P_D(j) \subset E\} = v^c(D)(E)$ for all $E \subset N$.

Thus $\sigma(A) \in Core(v^c(D))$.

Thus we have proved that

$$M(D) \subset \Sigma(D) \subset Core(v^c(D)). \tag{2}$$

Since $v^c(D)$ is convex it holds that $Core(v^c(D)) = Conv(M(D))$. With (2) it then follows that $Core(v^c(D)) = Conv(\Sigma(D))$.

□

Note that the number of permutations of the players in N is equal to $\#\Pi(N) = (\#N)!$, while the number of single predecessor digraphs in digraph D is equal to $\#\mathcal{A}_D = \prod_{i \in S(N)} \#P(i)$. Therefore, for many digraphs the number of single predecesor digraphs will be less than the number of permutations of N. However, this does not hold for all digraphs as can easily be seen by considering the *complete* digraph $D = N \times N$.

Example 3.2 Reconsider the digraph D and corresponding conservative score game v of Example 2.3.
This digraph has 4 single predecessor digraphs:

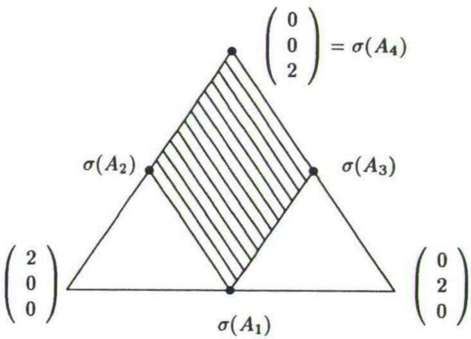

Figure 1: The core of v of Example 3.2

$$A_1 = \{(1,2),(2,4)\},\ A_2 = \{(1,2)(3,4)\},\ A_3 = \{(3,2),(2,4)\},\ A_4 = \{(3,2),(3,4)\}.$$

The corresponding score measures are:

$$\sigma(A_1) = (1,1,0,0),\ \sigma(A_2) = (1,0,1,0),\ \sigma(A_3) = (0,1,1,0),\ \sigma(A_4) = (0,0,2,0).$$

The core of v is represented by the shaded area in Figure 1. Note that the vectors $\sigma(A_i)$, $A_i \in \mathcal{A}_D$, are precisely the extreme points of the core.

According to relation (2) in the proof of Theorem 3.1 each extreme point of the core of a conservative score game corresponds to the score measure of a single predecessor digraph in the corresponding digraph. In Example 3.2 the converse also holds. However, this is not always the case as the following example shows.

Example 3.3 Consider the digraph $D = \{(1,2),(1,5),(2,4),(2,5),(3,2),(3,4)\}$ on $N = \{1,\ldots,5\}$ as represented in Figure 2.
This digraph has 8 single predecessor digraphs which are given by

$$A_1 = \{(1,2),(1,5),(2,4)\},\ A_2 = \{(1,2),(1,5),(3,4)\},\ A_3 = \{(1,2),(2,5),(2,4)\},$$

$$A_4 = \{(1,2),(2,5),(3,4)\},\ A_5 = \{(3,2),(1,5),(2,4)\},\ A_6 = \{(3,2),(1,5),(3,4)\},$$

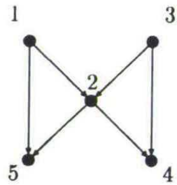

Figure 2: Digraph D of Example 3.3

$$A_7 = \{(3,2),(2,5),(2,4)\}, \quad \text{and } A_8 = \{(3,2),(2,5),(3,4)\},$$

with score measures

$$\sigma(A_1) = (2,1,0,0,0), \ \sigma(A_2) = (2,0,1,0,0), \ \sigma(A_3) = (1,2,0,0,0),$$

$$\sigma(A_4) = (1,1,1,0,0), \ \sigma(A_5) = (1,1,1,0,0), \ \sigma(A_6) = (1,0,2,0,0),$$

$$\sigma(A_7) = (0,2,1,0,0), \quad \text{and } \sigma(A_8) = (0,1,2,0,0).$$

According to Lemma 2.2 the corresponding conservative score game is given by $v = u_{\{1,2\}} + u_{\{1,3\}} + u_{\{2,3\}}$. Thus

$$M(D) = \{(2,1,0,0,0),(2,0,1,0,0),(1,0,2,0,0),(1,2,0,0,0),(0,2,1,0,0),(0,1,2,0,0)\}.$$

The core of v and the score measures of the single predecessor digraphs in D are illustrated in Figure 3. (The core corresponds to the shaded area in the figure.)
Consider the score measure $\sigma(A_4) = \sigma(A_5) = (1,1,1,0,0)$. This score measure does not correspond to a marginal vector of v. The intuition behind this is as follows. Each pair of the players 1, 2 and 3 has a different successor in common. Therefore in any permutation of the players the marginal contribution of the first of the three players 1, 2 or 3, to enter is equal to 0, of the second to enter is equal to 1, and of the third to enter is equal to 2. Thus there is no permutation in which the marginal contribution of each of these three players is equal to 1.

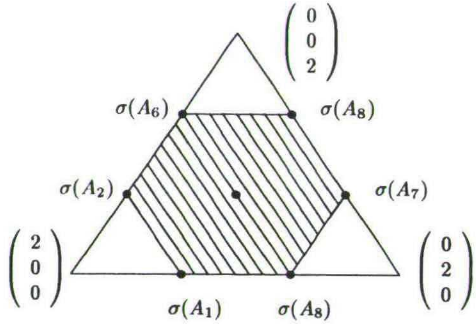

Figure 3: The core of v of Example 3.3

Next we will characterize the class of digraphs D for which the set $\Sigma(D)$ of score measures of all single predecessor digraphs coincides with the set $M(D)$ of marginal vectors of the corresponding conservative score game. For this we introduce the following concept.

Definition 3.4 *Let $D \in \mathcal{D}^N$. A sequence of nodes $(i_1, j_1, i_2, j_2, \dots, i_t, j_t)$, $t \geq 2$, is an* **anti-directed semi-circuit** *in D if the following conditions are satisfied*

(i) $i_k \neq i_l$ *and* $j_k \neq j_l$ *for all* $k, l \in \{1, \dots, t\}$ *with* $k \neq l$;

(ii) $j_k \in S(i_k) \cap S(i_{k+1})$ *for all* $k \in \{1, \dots, t-1\}$ *and* $j_t \in S(i_t) \cap S(i_1)$.

Thus an anti-directed semi-circuit in a digraph D is a cycle such that the neighbours of each node i in the cycle are either both successors or both predecessors of i and, moreover, each node appears at most once as a successor and at most once as a predecessor in this cycle. In the digraph of Example 3.3 the sequence $(1, 2, 3, 4, 2, 5)$ is an anti-directed semi-circuit. In the digraph of Examples 2.3 and 3.2 there does not exist an anti-directed semi-circuit.

Theorem 3.5 *Let $D \in \mathcal{D}^N$. Then*

$\Sigma(D) = M(D)$ *if and only if there is no anti-directed semi-circuit in D.*

The proof of this theorem can be found in the appendix and is more or less based on the reasoning as given in Example 3.3.

4 The Shapley value of a conservative score game

In this section we consider the Shapley value of a conservative score game. By definition the Shapley value is the function $Sh: \mathcal{G}^N \to \mathbb{R}^N$ which assigns to each TU-game the average of the marginal vectors of that game. In particular, for every $D \in \mathcal{D}^N$ it holds that

$$Sh_i(v^c(D)) = \frac{1}{\#M(D)} \sum_{m \in M(D)} m_i \text{ for all } i \in N. \tag{3}$$

A *power measure* for digraphs is a function $f: \mathcal{D}^N \to \mathbb{R}^N$ that assigns a real number to each node of a digraph which can be seen as a measure of the 'dominance power' of that node in the digraph. The score measure can be seen as such a power measure. Another power measure, introduced in van den Brink and Gilles (1992), is the *BG-measure* $\beta: \mathcal{D}^N \to \mathbb{R}^N$, which is given by

$$\beta_i(D) = \sum_{j \in S(i)} \frac{1}{\#P(j)} \text{ for all } i \in N \text{ and } D \in \mathcal{D}^N. \tag{4}$$

In section 2 we argued that the conservative score mapping assigns to every digraph a TU-game which can be seen as a social power measure that measures the dominance power of coalitions of nodes in the underlying digraph. Since the Shapley value assigns real numbers to all players in a TU-game, the Shapley value as expressed in equation (3) can be seen as a power measure for digraph D. It turns out that the Shapley value of a conservative score game is equal to the BG-measure of the underlying digraph. Moreover, this Shapley value, which by convexity of the game is the barycenter of the core (Shapley (1971)), is equal to the average of the score vectors corresponding to the single predecessor digraphs in the digraph. This is a rather surprising result since Example 3.3 shows that this set of score vectors may contain the extreme points of the core as a strict subset.

Theorem 4.1 *Let $D \in \mathcal{D}^N$. Then $Sh(v^c(D)) = \beta(D)$ and*

$$Sh_i(v^c(D)) = \frac{1}{\#\mathcal{A}_D} \sum_{A \in \mathcal{A}_D} \sigma_i(A) \text{ for all } i \in N.$$

PROOF

Let $D \in \mathcal{D}^N$. According to Lemma 2.10 the conservative score game $v^c(D)$ is the dual of the score game $v^s(D)$. In van den Brink and Gilles (1992) it is shown that $Sh(v^s(D)) = \beta(D)$. Since the Shapley value of a TU-game is equal to the Shapley value of its dual game it holds that $Sh(v^c(D)) = \beta(D)$.

For every $(i,j) \in D$ we define $\mathcal{A}_D(i,j) := \{A \in \mathcal{A}_D \mid j \in S_A(i)\}$, being the collection of single predecessor digraphs in D in which i is the (unique) predecessor of j.
Then $\#\mathcal{A}_D(i,j) = \prod_{h \in S(N)\setminus\{j\}} \#P(h)$. Since $\#\mathcal{A}_D = \prod_{h\in S(N)} \#P(h)$ it then holds that[†]

$$\beta_i(D) = \sum_{j \in S_D(i)} \frac{1}{\#P(j)} = \frac{1}{\#\mathcal{A}_D} \sum_{j\in S_D(i)} \#\mathcal{A}_D(i,j) =$$

$$= \frac{1}{\#\mathcal{A}_D} \sum_{j\in S_D(i)} \sum_{A \in \mathcal{A}_D(i,j)} 1 = \frac{1}{\#\mathcal{A}_D} \sum_{A\in\mathcal{A}_D} \sum_{j \in S_A(i)} 1 = \frac{1}{\#\mathcal{A}_D} \sum_{A\in \mathcal{A}_D} \sigma_i(A).$$

□

Example 4.2 Consider the digraph D of Example 3.3. For this digraph it holds that $Sh(v^c(D)) = \beta(D) = (1,1,1,0,0)$. Taking the average of the marginal vectors over all permutations of the players yields

$$\frac{1}{120}\left(\begin{array}{c} (20(2,1,0,0,0)+20(2,0,1,0,0)+20(1,0,2,0,0) \\ \\ +20(1,2,0,0,0)+20(0,2,1,0,0)+20(0,1,2,0,0) \end{array}\right) = (1,1,1,0,0),$$

while taking the average of the score measures of all single predecessor digraphs in D yields

$$\frac{1}{8}\left(\begin{array}{c} (2,1,0,0,0)+(2,0,1,0,0)+(1,0,2,0,0)+(1,1,1,0,0) \\ \\ +(1,1,1,0,0)+(1,2,0,0,0)+(0,2,1,0,0)+(0,1,2,0,0) \end{array}\right) = (1,1,1,0,0).$$

[†]This part is similar to the proof that the BG-measure of an acyclic, quasi-strongly connected digraph D is equal to the average of the score measures over all single predecessor digraphs in D as shown in van den Brink and Gilles (1993).

There are various axiomatizations of the Shapley value for arbitrary TU-games. One of these axiomatizations uses the efficiency, dummy player, necessary player and additivity axioms. A function $f: \mathcal{G}^N \to \mathbb{R}^N$ is *efficient* if $\sum_{i \in N} f_i(v) = v(N)$ for all $v \in \mathcal{G}^N$. It satisfies the *dummy player* property if $f_i(v) = 0$ for all $i \in N$ and $v \in \mathcal{G}^N$ with $v(E) = v(E \setminus \{i\})$ for all $E \subset N$. It satisfies the *necessary player* property if for all *monotone* games[‡] v and $i \in N$ such that $v(E) = 0$ for all $E \subset N \setminus \{i\}$ it holds that $f_i(v) \geq f_j(v)$ for all $j \in N$. It is *additive* if for all $v, w \in \mathcal{G}^N$ it holds that $f(v) + f(w) = f(v + w)$, where $(v + w)(E) = v(E) + w(E)$ for all $E \subset N$. In order to axiomatize the Shapley value restricted to the class of conservative score games we need to adapt the additivity axiom since for two conservative score games v and w the sum game $(v + w)$ need not to be a conservative score game as the following example shows.

Example 4.3 Let $N = \{1, 2, 3\}$, $v = u_{\{1\}} + u_{\{1,2\}}$, and $w = u_{\{1,3\}}$. The (unique) digraphs underlying these conservative score games are

$$D^1 = \{(1,2), (1,3), (2,3)\} \text{ and } D^2 = \{(1,2), (3,2)\}.$$

Using Proposition 2.9 it can be seen that no digraph can be constructed that underlies the sum game $(v + w) = u_{\{1\}} + u_{\{1,2\}} + u_{\{1,3\}}$.

We say that a function $f: \mathcal{G}^N \to \mathbb{R}^N$ is additive on a specific subclass $\mathcal{G}$ of $\mathcal{G}^N$ if for every pair of games $v, w \in \mathcal{G}$ such that the sum game $(v + w)$ is also an element of $\mathcal{G}$ it holds that $f(v + w) = f(v) + f(w)$. Now the Shapley value restricted to the class of conservative score games can be axiomatized by the efficiency, dummy and necessary player axioms and additivity restricted to the class of conservative score games. (This can be seen by noting that each unanimity game, except the unanimity game on N, is a conservative score game and each conservative score game can be expressed as in Lemma 2.2.)

As stated in Theorem 4.1, the BG-measure of a digraph D is equal to the Shapley value of the conservative score game $v^c(D)$. It is interesting to note that by combining the set

[‡]A TU-game v is monotone if $v(E) \leq v(F)$ for all $E \subset F \subset N$.

of axioms that characterize the conservative score mapping and the set of axioms that characterize the Shapley value restricted to the class of conservative score games one obtains a set of axioms that uniquely determine the BG-measure as a power measure $f: \mathcal{D}^N \rightarrow \mathbb{R}^N$. For example, the dummy node properties of the conservative score mapping and the Shapley value, respectively, state that a node that does not dominate any other node in a digraph is a dummy player in the corresponding game, and that the Shapley value of a dummy player in a game is equal to 0. Combining these two axioms yields the dummy node property of the BG-measure which states that the BG-measure of a node that does not dominate any other node is equal to 0. Similarly, combining the other three properties yield the corresponding properties for the BG-measure. As can be derived from van den Brink and Gilles (1992) these four properties characterize the BG-measure.

Appendix

Before proving Theorem 3.5 we introduce the following.
An anti-directed semi-circuit $(i_1, j_1, \ldots, i_t, j_t)$ is **minimal** in D if there is no anti-directed semi-circuit $(h_1, g_1, \ldots, h_s, g_s)$ in D such that $\{h_1, \ldots, h_s\} \subset \{i_1, \ldots, i_t\}$.

PROOF OF THEOREM 3.5
Let $D \in \mathcal{D}^N$. Since $M(D) \subset \Sigma(D)$ (as shown in the proof of Theorem 3.1) we have to prove that $\Sigma(D) \subset M(D)$ if and only if there is no anti-directed semi-circuit in D.

Only if
Suppose there is an anti-directed semi-circuit in D. Then there is a minimal anti-directed semi-circuit $(i_1, j_1, \ldots, i_t, j_t)$ in D.
Let $I = \{i_1, \ldots, i_t\}$ and $J = \{j_1, \ldots, j_t\}$.
We distinguish the following two cases with respect to $\#I$.

1. Suppose that $\#I = 2$, i.e., $I = \{i_1, i_2\}$.

   Then we can construct an $A \in \mathcal{A}_D$ that satisfies the following conditions:

   (i) $P_A(j_1) = i_1$ and $P_A(j_2) = i_2$;

(ii) $P_A(h) \subset P_D(h) \cap I$ for all $h \in S(I) \setminus J$.

Since $j_1 \notin S(i_1) \setminus S(i_2)$ and $j_2 \notin S(i_2) \setminus S(i_1)$ it holds that

$$\sigma_{i_1}(A) \geq \#[S(i_1) \setminus S(i_2)] + 1 \text{ and } \sigma_{i_2}(A) \geq \#[S(i_2) \setminus S(i_1)] + 1.$$

Let $\pi \in \Pi(N)$. We may assume without loss of generality. that $\pi(i_1) < \pi(i_2)$.

Then $m^{\pi}_{i_1}(v^c(D)) \leq \#[S(i_1) \setminus S(i_2)] < \#[S(i_1) \setminus S(i_2)] + 1$.

Hence, there is no $\pi \in \Pi(N)$ such that $\sigma(A) = m^{\pi}(v^c(D))$.

2. Suppose that $\#I > 2$.

   (a) Let $h \in S(I) \setminus J$. We first prove that $\#[P(h) \cap I] = 1$.
   Since $h \in S(I)$ it holds that $\#[P(h) \cap I] \geq 1$.
   Suppose that $\#[P(h) \cap I] > 1$. Then there exist $k, l \in \{1, \ldots, t\}$ with $k < l$ and $\{i_k, i_l\} \subset P(h)$. We may assume without loss of generality. that $k = 1$.
   If $l < t$ then $(i_1, j_1, \ldots, i_l, j_l)$ is an anti-directed semi-circuit in D with $\{i_1, \ldots, i_l\} \subset I$ and $\{i_1, \ldots, i_l\} \neq I$.
   If $l = t$ then (i_1, j_1, i_t, j_t) is an anti-directed semi-circuit in D with $\{i_1, i_t\} \subset I$ and $\{i_1, i_t\} \neq I$ (since by assumption $\#I > 2$).
   Both cases are in contradiction with $(i_1, j_1, \ldots, i_t, j_t)$ being a minimal anti-directed semi-circuit in D.

   (b) Using (a) we can construct an $A \in \mathcal{A}_D$ that satisfies the following conditions:

      (i) $P_A(j_k) = \{i_k\}$ for all $k \in \{1, \ldots t\}$;

      (ii) $P_A(h) = P_D(h) \cap I$ for all $h \in S(I) \setminus J$.

   (c) Next we prove that $\sigma(A) \notin M(D)$.
   Since $S_A(i_k) = S_D(i_k) \setminus \{j_{k-1}\}$ for all $k \in \{2, \ldots, t\}$ and $S_A(i_1) = S_D(i_1) \setminus \{j_t\}$, it holds that

$$\sigma_i(A) = \#S_D(i) - 1 \text{ for all } i \in I. \tag{5}$$

Let $\pi \in \Pi(N)$ and let $i \in I$ be such that $\pi(i) \leq \pi(h)$ for all $h \in I$. Then $\{j \in S_D(i) \mid P_D(j) \subset \{h \in N \mid \pi(h) \leq \pi(i)\}\} \cap J = \emptyset$.

But then

$$m_i^\pi(v^c(D)) = \#\{j \in S_D(i) \mid P_D(j) \subset \{h \in N \mid \pi(h) \leq \pi(i)\}\} \leq \#S_D(i) - 2.$$

Hence there is no $\pi \in \Pi(N)$ such that $\sigma(A) = m^\pi(v^c(D))$.

Thus we have proved that if there is an anti-directed semi-circuit in D then $\Sigma(D) \not\subset M(D)$.

If

Suppose there is no anti-directed semi-circuit in D, and let $A \in \mathcal{A}_D$. We show that there is a $\pi \in \Pi(N)$ such that $\sigma(A) = m^\pi(v^c(D))$.

(a) We recursively define the sets $L_k \subset N$, $k \in \mathbb{N} \cup \{0\}$, as follows:

$$L_0 := \emptyset$$

and

$$L_k := \left\{ i \in N \setminus \bigcup_{l=0}^{k-1} L_l \;\middle|\; \begin{array}{c} \text{For every } j \in S_A(i) \text{ it holds that} \\ P_D(j) \subset \bigcup_{l=0}^{k-1} L_l \cup \{i\} \end{array} \right\} \text{ for all } k \in \mathbb{N}.$$

(b) Let $k \in \mathbb{N}$ be such that $N \setminus \bigcup_{l=0}^{k-1} L_l \neq \emptyset$. We prove that $L_k \neq \emptyset$.

On the contrary suppose that $L_k = \emptyset$. Let $\widehat{N} := N \setminus \bigcup_{l=0}^{k-1} L_l$. Then for every $i \in \widehat{N}$ there is a $j \in S_A(i)$ such that $P_D(j) \not\subset \bigcup_{l=0}^{k-1} L_l \cup \{i\}$ and thus $[P_D(j) \cap \widehat{N}] \setminus \{i\} \neq \emptyset$.

Let $\hat{n} = \#\widehat{N}$. We can construct a sequence $(i_1, j_1, i_2, j_2, \ldots)$ satisfying the following conditions:

(i) $i_1 \in \widehat{N}$;

(ii) $j_k \in S_A(i_k)$ and $i_{k+1} \in [P_D(j_k) \cap \widehat{N}] \setminus \{i_k\}$ for every $k \in \{1, \ldots \hat{n}\}$.

Since $\widehat{N}$ is finite there are $k, l \in \{1, \ldots, \widehat{n}+1\}$ with $k < l$, $i_k = i_l$ and all players in $\{i_k, \ldots, i_{l-1}\}$ different.

By construction it holds that $j_r \neq j_s$ for all $r, s \in \{k, \ldots, l-1\}$.

But then $(i_k, j_k, \ldots, i_{l-1}, j_{l-1})$ is an anti-directed semi-circuit in D.

(c) From (a) and (b) it follows that there is an $m \in \mathbb{N}$ such that

(i) $L_k \neq \emptyset$ for all $k \in \{1, \ldots, m\}$;

(ii) $L_k \cap L_l = \emptyset$ for all $k, l \in \{1, \ldots, m\}$ with $k \neq l$;

(iii) $\bigcup_{k=1}^{m} L_k = N$,

i.e., the sets $L_1, \ldots, L_m$ form a partition of N consisting of non-empty sets only.

(d) Let $\pi \in \Pi(N)$ be such that if $i \in L_k$ and $j \in L_l$ with $k < l$ then $\pi(i) < \pi(j)$. We show that $m^{\pi}(v^c(D)) = \sigma(A)$.

Let $i \in L_k$ for some $1 \leq k \leq m$. From (a) it follows that

(i) If $j \in S_A(i)$ then $P_D(j) \subset \bigcup_{l=0}^{k-1} L_l \cup \{i\}$;

(ii) If $j \in S_D(i) \setminus S_A(i)$ and $S_A(h) = \{j\}$ then $h \in \bigcup_{l=k+1}^{m} L_l$ and thus $P_D(j) \not\subset \bigcup_{l=0}^{k-1} L_l \cup \{i\}$.

But then

$$\begin{aligned} m_i^{\pi}(v^c(D)) &= \#\{j \in S_D(i) \mid \pi(h) \leq \pi(i) \text{ for all } h \in P_D(j)\} \\ &= \#\left\{ j \in S_D(i) \,\middle|\, P_D(j) \subset \bigcup_{l=0}^{k-1} L_l \cup \{i\} \right\} = \sigma_i(A). \end{aligned}$$

Thus we have proved that if there is no anti-directed semi-circuit in D then $\Sigma(D) \subset M(D)$.

$\square$

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